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STUDY OF CLASSIFICAITON OF BIOTOPOLOGICAL SPACE AND ITS PROPERTIES

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Abstract

J. C. Kelley introduced the concept of bitopoloical spaces which are being discussed with suitable examples. We discuss here the concepts of pairwise continuous mapping, pairwise open mapping and pairwise homeomorphism of bitopological spaces to distinguish each from other. We analyse comparative merits and demerits different types of classifications of bitopological space.

Keywords:- bitopoloical, open mapping, homeomorphism, Quasi-pseudo metric, non empty set, open subsets,

Introduction

We explain the existing pariwise (T_0), (T_1) and (T_2) concept and derive their properties. The concepts of Stp-compactness, p-compactness and Sgp-compactness for bitopologcal spaces are examined to compare with the existing concepts of pairwise compactness for bitopological space. We derive some properties of connectedness for regular and Normal spaces. We discuss some basic concepts of importance in course of further discussion.

(i) Quasi Pseudometric structure

Let X be a non empty set A non-negative real valued function p(,) defined on X × X is called a quasi-pseude-metric on X if the following conditions are satisfied some examples are given below.

(i) $P(x, x) = 0 \quad (X \in X)$

(ii) $p(x, z)) \leq p\ (x, y) + p\ (y, z) \quad (x, y, z \in X)$

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If, in addition, p (,) also satisfies the condition

(iii) $p(x, y) = 0$ iff $x = y$

Then p(,) is called a quasi-metric on X. We thus find that a quasi – metric p(,) on X is a metric on X if $p(x, y) = p(y, x)$, for all $x, y \in X$. The alternative approach to introduce has been developed with suitable examples.

(ii) Quasi-pseudo metric

For a given quasi-pseudo metric p(,) on X, a function q(,) may be defined on X x X as follows

$q(, x, y) = p(y, x) \quad (x, y \in X)$ …(1.1)

then q(,) is also a quasi-pseudo-metric on X, and it is said that p(,) and q(,) are conjugate to each other. It is known that every quasi-pseudo-metric p(,) on X gives rise to a topology on X which is presented in the following way

Let $\delta_p(x, r) = \{y \in X : p(x, y) < r\}$ ….(1.2)

be a p-open sphere with centre at x and radius $r > 0$ with respect to quasi-pseudo-metric p(,). The collection of all p-open spheres from an open base for a topology T_1 on X and such topology on X is called the topology determined by p(,).

(iii) Definition (p-open)

An open subset under this topology is called p-open subset or a T_1-open set. Similarly, q(,) also determines a topology on X denoted by T_2 and an open subset under this topology is called q-open subset of X or a T_2 – open set.

Let $X = \{a, b, c\}$ be a non-empty set

(a) Let p(,) be defined on X x X as follows.

$P(a, a) = 0, p(b, a) = 0, p(c, a) = 0$
$P(a, b) = 1, p(b, b) = 0, p(c, b) = 0$ …(1.3)

P(a, c) = 1, p(b, c) = 0, p (c, c) = 0

Then p(,) is a quasi –pseudo – metric on X.

Also, for a real r > 1, we have

δ_p (a, r) = {x : p (a, x) < r} = {a}

δ_p (b, r) = {x : p (b, x) < r} = {a, b, c} = X …(1.4)

δ_p (c, r) = {x : p (c, x) < r} = {a, b, c} = X

So the topology T_1 generated by p(,) on X is the topology generated by the family {{a}, X},

i.e., T_1 = {∅, {a}, X} …(1.5)

The following relations are defined as follows.

q(a, a) = 0, q(b, a) = 1 , q(c, a) = 1
q(a, b) = 0, q(b, b) = 0 , q(c, b) = 0
q(a, c) = 0, q(b, c) = 0 , q(c, c) = 0

Case-ii) Now for r < 1, we have

δ_q (a, r) = {x : q (a, x) < r} = {a, b, c} = X
δ_q (b, r) = {x : q (b, x) < r} = {b, c} …(1.6)
δ_q (c, r) = {x : q (c, x) < r} = {c, b}

Here the topology generated by q(,) on X is

T_2 = {∅, {b, c}, X} …(1.7)

Thus, we find that p(,) and q(,) give rise to two topologies T_1 and T_2 on X respectively.

Definition (Bitopological space)

A bitopological space is a set X together with two topologies T_1 and T_2 defined on X and it is denoted by (X, T_1, T_2).

The study of bitopological space which are generated by T_1 and T_2 are known to be conjugate, Quasi-pseudo-metrics p(,) and q(,). However, the theory for biotopological spaces in which associated topologies are arbitrary and also come within their scope bitopological spaces.

Complementary Bitopological spaces

Let (X, T_1, T_2) be a bitopological space and let A $\subset$ x. Then (A, T_{1A}, T_{2A}) is called an induced bitopological space where T_{1A} and T_{2A} are induced topological in A with respect to T_1 and T_2 respectively. If (X, T_1, T_2) is bitopological space and A $\subset$ X, then by a sub bitopological space A. A bitopological space (X, T_1, T_2) is said to be complementary if G is open in T_1 $\Leftrightarrow$ G^c is open in T_2. We derive some properties of complementary bitopological space.

Theorem

Let (X, T_1, T_2) be a bitopological space in which T_1 and T_2 are topologies generated by pair of conjuatate quasi-pseude-metrics. The (X, T_1, T_2) is a complementary bitopological space if the range of a quasi-pseudo-metric is finite.

Proof

Let us suppose that the topologies T_1 and T_2 are generated by a pair of conjugate quasi-pseudo –metrics p(,) and q(,) respectively. suppose that the range of p(,) is finite. Let A be a p-open base in X. then there are exists a $\in$ X and r > 0 such that

A = {s: p(a, x < r)} ...(1.8)

This means that y $\in$ A^c iff p(a, y) $\geq$ r. Since the range of p(,) is finite, we see that

$$\underset{X \in A}{Max}\ p(a, x) = 1 < r \ and \ \underset{X \notin A}{min}\ p(a, x) = 1' \geq r \qquad ...(1.9)$$

$\Rightarrow$ 1, < 1′ and so 1 < $\frac{1+1'}{2}$ < 1. We put m = $\frac{1+1'}{2}$. Thus we have

P(a, x) < m, X $\in$ A and p(a, x) > m, x $\in$ A^c. for each b $\in$ A^c, we have p(a, b) = k > m. Let

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$M' = k - m$ and let

$B = \{x : q(b, x) < m'\} = \{x : p(x, b) < m'\}$

Now for $x \in B$, we have

$P(a, b) \leq p(a, x) + p(x, b)$ ...(1.10)

$\Rightarrow k < p(a, x) + m'$

$\Rightarrow k - m' < p(a, x)$

$\Rightarrow m < p(a, x)$

$\Rightarrow p(a, x) > m$

$\Rightarrow x \in A^c$

So $B \subset A^c$

In other words with each $b \in A^c$, we get q - n′ hood of b which is subset of A^c, so A^c is union of q-open sets. this implies that A^c is q – open in X. By considering a p-open subset G of X, then it is union of p-basic open sets, whose complements are q – open in X. it implies that G^c is q – open in X (i.e., G^c is open in T_2). Similarly it may be proved that if H is open in T_2, then H^c is open in T_1. Hence, (X, T_1, T_2) is a complementary bitopological space.

Pairwise continuity, pairwise openness and pairwise homeomorphism

A mapping f of a bitopological space (X, T_1, T_2) into a bitopological space (Y, T_3, T_4) is said to be continuous iff the induced mappings $f_1 : (X, T_1) \rightarrow (Y, T_3)$ and $f_2 : (X, T_2) \rightarrow (Y, T_4)$ of the topological spaces are continuous.

Seperation axoms of Bitopological spaces

i)The first classification is based on whether the two topologies T_1 and T_2 are generated by a pair of conjugate pair or not.

ii)The second classification is based on whether $T_1 \cap T_2 = \{\varnothing, X\}$ or $T_1 \cup T_2$ is a topology.

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iii)The third classification is based on whether $T_1 \subseteq T_2$ or $T_2 \subseteq T_1$.

Interconnections between (T_0), (T_1) and (T_2) space (T_3) and (T_4).

Theorem

A bitopological space (X, T_1, T_2) of class (ii) is a complementary bitopological space if the ranges of associated quasi-pseudo-metrics are finite. Let us classify bitopological space under different restrictions.

A bitopological space (X, T_1, T_2) is said to be of type 4 if $T_1 \cup T_2$ is also a topology on X. The following characteristic classifications of bitopological space are stated.

i)Class of all bitopological spaces which are neither of type T_3 nor of type – T_4

ii)Class of all bitopological spaces which are of type T_3 but not of type – T_4

iii)Class of all bitopological spaces which are of type T_4 but not of type – T_3

iv)Class of all bitoopological spaces which are neigher of type T_3 and also type – T_4 (both)

Topological properties of bitopological spaces

(a) A pair of topological space (X, T_1) and (X, T_2)

(b) A topological space $(X, T_1 + T_2)$ where $T_1 + T_2$ is the topology generated by T_1 U T_2.

Let P be a topological property. Then we say that (X, T_1, T_2) is

(i) p-p if (X, T_1, T_2) is pairwise P :

(ii) bi-p if both (X, T_1) and (X, T_2) have property P

(iii) stp – p if (X, T_1, T_2) is pairwise p but neither (X, T_1) nor (X, T_2) have property P

(iv) wp-p if (X, T_1, T_2) has some weak pairwise property P

(v) sup- P if $(X, T_1 + T_2)$ has propoerty P;

(vi) bitopological P if it remains invariant under homeomorphism (bitopological) and

(vii) p-bitopological p if it remains invariant under pairwise homeromorhism.

The modified versions are as follows :

(i) (strictly pairwise (T_0)

A bitopological space (X, T_1, T_2) is said to be strictly pairwise (T_o) – space if neither (X, T_1) nor (X, T_2) is (T_0) – space but for every pair of distinct points of X, there exists either a T_1-open set or a T_2 – open set containing one of them but not the other.

(ii) A bitopological space (X, T_1, T_2) is said to be pairwise compact if the topological space (X, T_1) nor (X, T_2) are compact. Let us illustrate it by a suitable example.

(iii) A collection U of either T_1-open or T_2 – open sets with $X \subseteq$ u U is called

(i) stp-open cover of X if

$(U \cap T_1) - (U \cap T_2) \neq \varnothing$ and $(U \cap T_2) - (U \cap T_1) \neq \varnothing$

(ii) p-open conver of X if

$(U \cap T_1) \neq \varnothing$, $(U \cap T_2) \neq \varnothing$

(iii) stp – open cover of X, if $U \subseteq T_1 \cup T_2$

(iv) A bitopological space (X, T_1, T_2) is said to be

(i) stp-compact if every stp-open cover of X has finite stp –open subcover

(ii) p-compact if every p-open cover of X has finite p-open subcover

(iii) sgp-compact if very sgp –open cover of X has finite sgp –open subcove

(v) A bitopological space (X, T_1, T_2) is said to be

(i) stp-Lindeloff if every stp-open cover of X has countable stp –open subcover

(ii) p-Lindeloff if every p-open cover of X has countable p-open subcover

(iii) sgp-Lindeloff if very sgp –open cover of X has countable sgp –open subcover

Properties of strictly pair-wise compactness and its equivalence.

(i) Stp-compactness $\nRightarrow$ pairwise compactness

(ii) p-compactness $\nRightarrow$ stp- compactness

(iii) p-compactness $\nRightarrow$ pairwise- compactness

(iv) pairwise compactness $\nRightarrow$ p- compactness

(v) pairwise compactness $\nRightarrow$ sgp- compactness

(vi) stp compactness $\nRightarrow$ sgp- compactness

(vii) p-compactness $\nRightarrow$ sgp- compactness

Theorem

Let (X, T_1, T_2) be sgp – compact bitopological space, then

(i) (X, T_1, T_2) is pairwise compact bitopological space

(ii) (X, T_1, T_2) is stp - compact bitopological space

(iii) (X, T_1, T_2) is p- compact bitopological space

Proof

Let us suppose that (X, T_1, T_2) is sgp-compact bitopological space. Then every sgp-open cove of X has a finite – sgp open sub-cover of X.

(i) Let U be a collection of T_1 – open subsets which covers X. Then $U \subseteq T_1 \cup T_2$. So U is a sgp – open cover of X. Hence, open cover has a finite sgp – open subcover whose elements are T_1 – open. This means that (X, T_1) is compact. Similarly we may prove that (X, T_2) is compact. Hence, X, T_1, T_2) is pairwise compact bitopological space.

(ii) Let U be a stp – open cover of X. Then U $\subseteq T_1 \cup T_2$. So U is a sgp – open cover of X. According to assumption, of theorem stated above has finite sgp – open subcover x say V. Let us choose $G_\alpha \in (U \cap T_2)$ and $G_\beta \in (U \cap T_2) - (U \cap T_1)$ then V U $\{G_\alpha, G_\beta\}$ is a finite step – open subcover of X. Hence, (X, T_1, T_2) is stp-compact bitopological space.

(iii) Let U be a p-open cover of X. Then U $\subseteq T_1 \cup T_2$. So U is a sgp – open cover of X. According to our assumption, U has a finite sgp – open subcover of X, say V. we choose $G_\alpha \in (U \cap T_1)$ and $G_\beta \in (U \cap T_2)$. Then V U $\{G_\alpha , G_\beta\}$ is a finite p – open subcover of X. Hence, (X, T_1, T_2) is p – compact bitopologial space.

References :-

1. Abd El-Monsef, M.E. Kozae, A.M. and Taher, B.M., (1997) Compactification in bitopological spaces, Arab J. for Sci. Engin., 22(1A), 99–105.

2. Arya, S.P. and Nour, T.M., (1988) Separation axioms for bitopological spaces, Indian J. Pure Appl. Math., 19(1), 42–50.

3. Kelly, J.C., (1963) Bitopological spaces, Proc. London Math. Soc., (3)13, 7–89.

4. Khedr, F.H. and Al-Shibani, A.M., (1991) On pairwise super continuous map- pings in bitoplogical spaces, Internat. J. Math. Math. Sci., (4)14, 715–722.

5. Misra, D., N., and Dube, K.K., (1973) Pairwise R0-spaces, Ann de la Soc. Sci, Bruxells, T. 87(1), 3–15.